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The effect of airflow cooling on a scramjet: a preliminary assessment

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THE ROYAL SOCIETY

The effect of airflow cooling on a scramjet: a preliminary assessment

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This paper gives a broad and simplified theoretical treatment of the effects on performance of a scramjet engine due to adding a precoolant to the airflow somewhere in the initial compression process. The study suggests that the largest increase in thrust can be derived from injection as far upstream as is practical. In particular, it can reduce the lowest Mach number at which a scramjet can usefully operate, although at some cost in propellant (i.e. fuel plus coolant) consumption. Liquid ammonia seems a particularly suitable airflow coolant, not least in that it can also release heat in downstream combustion.

Keywords: cooled compression scramjet; precooling in the intake; precooling through a shock; coolant and fuel flow; choice of coolant

1. Introduction

It is well known that the extraction of heat from the compression process of a propulsive cycle will boost the thrust of a given engine. In the early 1940s, Whittle (1953) demonstrated—in what were by report quite spectacular experiments—the effects of water and of ammonia addition on early turbojets, and Lundin (1949) has described the effects of water and water–alcohol. Roy (1946, 1958) is believed to have proposed the technique for ramjets, which are notoriously short of specific thrust especially at low flight Mach numbers, and Townend (1966) suggested the use of heat extraction from within the intake of a scramjet or the supersonic flow upstream.

The author is indebted to Townend for also suggesting the present study, which takes a first look at the possible advantages of precooling to the operation of a scramjet engine, especially at lower Mach numbers of around 4 or 5. It was felt that a simplified analysis of the engine performance would be sufficient for this purpose. The analysis and its assumptions are fully detailed in the appendix. In particular, it treats both combustion heating and precooling as if they were processes of external heat transfer, thereby escaping the need to tie them to the thermodynamics of particular reactions within the gas mixture. Further, the flow through the engine is assumed as simply that of a perfect gas with a constant specific heat ratio $\gamma = 1.4$, ignoring (for the sake of consistency) the mass addition to the air of fuel and coolant.

In what follows we use the analysis to look at two examples of precooling. The first, to be discussed in the next section, is the straightforward example of the introduction of a cryogen, such as liquid oxygen or liquid nitrogen, into the intake where it immediately vaporizes. This is supposed to take place ahead of the shock waves forming the compression system. The second example assumes injection of a liquid

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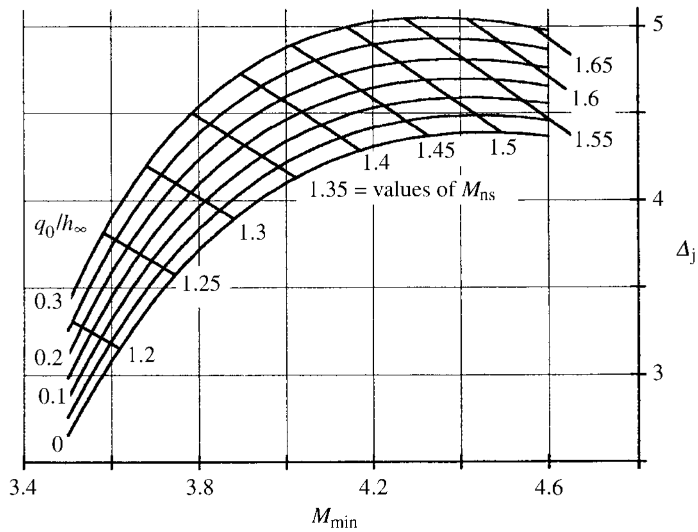

Figure 1. Effect of precooling (q_0/h_∞) of the intake flow on the kinetic energy increment Δ_j and the minimum flight Mach number M_{min} corresponding to choking, as determined by the incident shock Mach number M_{ns} of the four-shock compression (with $h_{max}/h_\infty = 10$).

with a higher boiling point (such as ammonia or maybe even water) that changes phase as it passes through one of the series of compression shocks. This is considered in some detail in § 3. Our conclusions are recorded in the final section.

2. Precooling in the intake

If the temperature, and so also the enthalpy, reached at the end of the combustion process is regarded as fixed, there are two distinct effects of upstream cooling. The jet velocity will be increased at any given flight speed, and the flight Mach number at which the flow is choked will be reduced. This latter is evidently also the minimum flight Mach number M_{min} at which the assumed engine performance is applicable.

This is illustrated in figure 1, which is composed on the assumption that the cooling takes place ahead of a four-shock compression and that the maximum enthalpy h_{max} reached at the completion of combustion is equal to 10 times the atmospheric enthalpy (h_∞). Because we assume a perfect gas, this is also equivalent to $T_{max} = 10T_\infty$, or since flight is supposed to take place in the stratosphere, it implies $T_{max} = 2166.5$ K. The figure shows the non-dimensional kinetic energy increment

$$\Delta_j = \tfrac{1}{2}(V_j^2 - V_\infty^2)/h_\infty. \tag{2.1}$$

The advantage of using Δ_j is that—according to our simplified model—it is independent of flight speed V_∞ and flight Mach number M_∞, for all $M_\infty \geqslant M_{min}$. Figure 1 shows the value of both Δ_j and M_{min} for different values of the normal Mach number M_{ns} across all the four compression shock waves, and for different amounts of precooling q_0 expressed as a fraction of h_∞. The jet velocity ratio can be readily calculated for all $M_\infty \geqslant M_{min}$ from the relation

$$\frac{V_j}{V_\infty} = \sqrt{1 + \frac{2\Delta_j}{(\gamma - 1)M_\infty^2}}. \tag{2.2}$$

Thus the higher the value of $\Delta_{\rm j}$, the higher will be the jet velocity at any given M_∞.

It will be seen that, with $M_{\rm ns}$ held fixed, increase of cooling increases $\Delta_{\rm j}$ and decreases $M_{\min}$. Decreasing $M_{\rm ns}$ causes $M_{\min}$ to become lower, whether or not there is precooling. On the other hand, there is an optimum $M_{\rm ns}$ that makes $\Delta_{\rm j}$ a maximum. This is close to $M_{\rm ns} = 1.49$ if there is no precooling, and increases to about 1.58 as the relative precooling q_0/h_∞ is increased to about 0.3. Below this optimum of $M_{\rm ns}$ there is a trade-off between the lower minimum Mach number and the lowered jet velocity ratio (as implied by $\Delta_{\rm j}$).

How could the values of q_0/h_∞ shown be achieved in practice? The actual numbers are sensitive to the pressure in the intake, but for *small* additions of either liquid nitrogen or liquid oxygen, q_0/h_∞ would be very roughly equal to 1.6μ, where μ is the ratio of the mass flow of the coolant to that of the airflow through the scramjet. Rather more effective cooling is available from cryogenics whose use would unfortunately be judged dangerous. For instance, for a small addition of liquid methane, q_0 is *ca.* $3.4\mu h_\infty$, while liquid hydrogen would be an even more effective coolant, yielding $q_0 \approx 15\mu h_\infty$. But if methane were used in a concentration much above $\mu > 0.1\%$, say, there would be a high risk of a damaging explosive reaction in contact with the air, and likewise with hydrogen even if μ were as low as 0.01%. Their role would have to be restricted to that of an exterior coolant, through the walls of the intake.

By far the most suitable additive coolant would seem to be liquid ammonia, for which $q_0 \approx 7\mu h_\infty$. Moreover, its boiling point (at small vapour pressures) is well below atmospheric temperature even in the stratosphere. We return to consider its use in § 3 *c*, where we suggest that it should be possible to evaporate a mass flow of over 3% of that of the air in the intake.

3. Precooling through an evaporative shock

Any unevaporated surplus of liquid ammonia, or a different coolant with a higher boiling point (water is the obvious example), would only evaporate after the air temperature has been sufficiently raised by passage through one or more of the compression shocks. If complete evaporation is achieved through just one evaporative shock then, by implication, the temperature of the incident flow would be at or below the boiling point of the coolant, $T_{\rm b}$ say. However, the air–vapour mixture leaving the shock would have a temperature above $T_{\rm b}$, which itself will have been increased a little by the increase of pressure through the shock. If complete evaporation is *not* achieved, then only that amount of coolant evaporates that is enough to lower the downstream temperature to equal the local boiling point.

It is therefore a necessary condition of any such evaporative shock that at least the temperature does not decrease through the wave. If we are willing to ignore the (small) change of $T_{\rm b}$ with pressure, as we shall do in what follows, then this is also a sufficient condition for the existence of such an evaporative shock, irrespective of how many there are.

We also assume in what follows that complete evaporation of the coolant is achieved through the one shock wave. Which shock it would be of a train of several in the compression would depend on the intake condition and the actual coolant used. However, in the interests of generality, we shall suppose that the boiling point can be selected at will. One might approach this condition notionally (if not in reality)

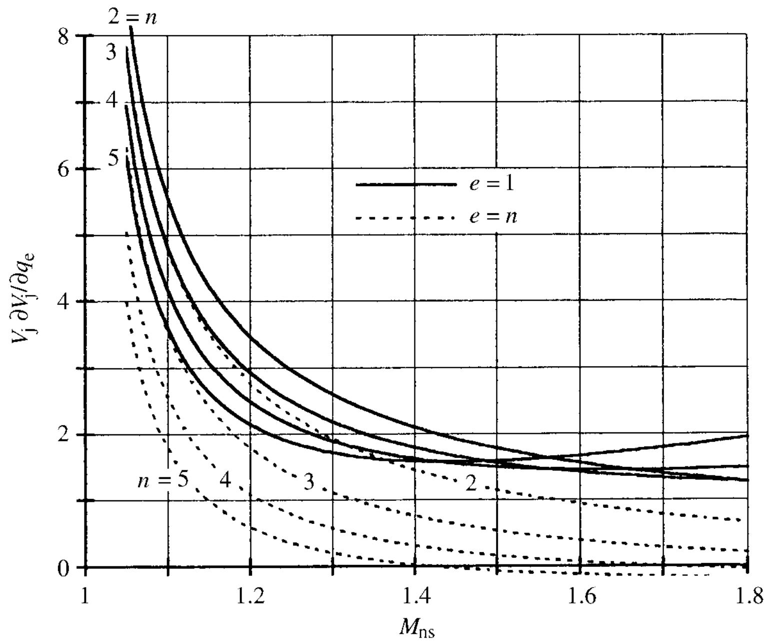

Figure 2. Effect on $V_j \partial V_j/\partial q_e$, the rate of change of jet kinetic energy with q_e, of placing the evaporative shock as the first ($e = 1$) or the last ($e = n$) of the n-shock train, as determined by the incident shock Mach number M_{ns} (with $h_{max}/h_\infty = 10$).

by supposing, for example, that the coolant was a mixture, such as a solution of ammonia in water.

(*a*) *The positioning of the evaporative shock*

The first question that becomes relevant is therefore, given the choice, through which shock of a sequence should a given loss of total heat by evaporation (q_e, say) take place? It appears that, at least if increase in jet velocity is the basis of choice and q_e is small, then it should be the first shock of the sequence.

This is exhibited in figure 2, which shows $V_j \partial V_j/\partial q_e$, the rate of change of jet kinetic energy with q_e, as a function of the incident Mach number M_{ns} used in the shock train, if the enthalpy after combustion is $h_{max} = 10h_\infty$. Results are shown for between $n = 2$ and 5 shocks with the evaporative shock (numbered as e in the sequence) being either the first ($e = 1$) or the last ($e = n$) in the succession. The greater effectiveness of the first position is further illustrated in figure 3, which again shows $V_j \partial V_j/\partial q_e$, but this time as a function of h_{max}/h_∞, and assuming that M_{ns} is chosen to maximize the jet kinetic-energy increment Δ_j as given in equation (2.1). In this particular instance, the value of $V_j \partial V_j/\partial q_e$ is simply a function of $(n - e)$, rather than of n and e separately. In particular, the almost complete ineffectiveness of evaporation occurring in the last shock of the sequence ($n - e = 0$) is quite clear.

The values of M_{ns} that maximize the jet kinetic energy increment Δ_j (and so maximize V_j at any given M_∞) without precooling are shown in figure 4 as a function of the number of shocks, n. Thus for $h_{max} = 10h_\infty$ and $n = 4$, the optimum M_{ns} is a little below 1.49. If we refer back to figure 1, it will be seen that this is indeed the

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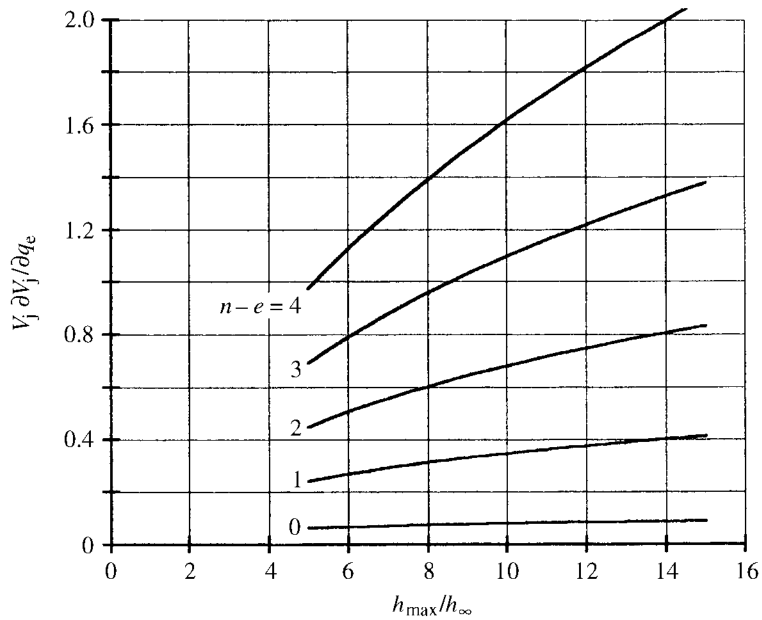

Figure 3. Effect on $V_j \partial V_j / \partial q_e$, the rate of change of jet kinetic energy with q_e, of the position e of the evaporative shock in an n-shock train, as determined by the ratio of the enthalpy after combustion h_{max} to that of the free-stream h_∞, assuming the incident shock Mach number M_{ns} is chosen to optimize Δ_j.

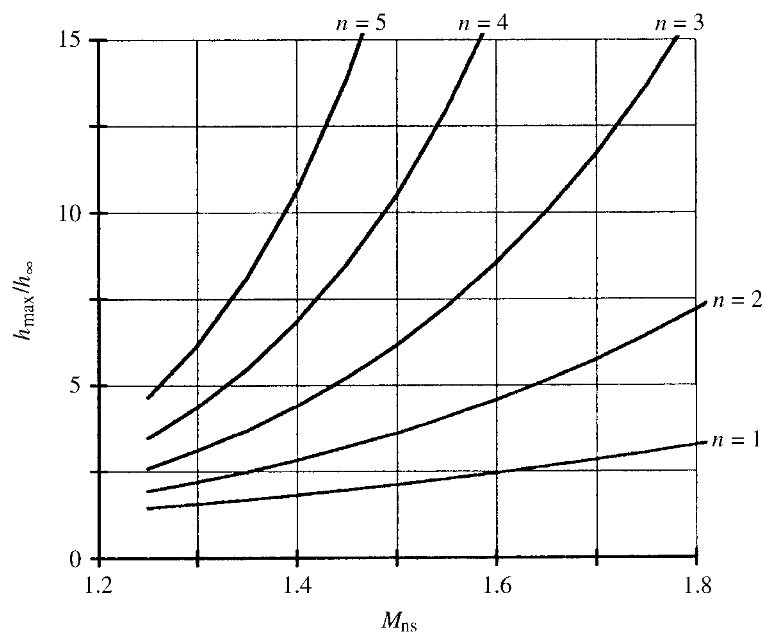

Figure 4. The relation between the enthalpy ratio h_{max}/h_∞ and the incident shock Mach number M_{ns} of an n-shock train, that optimizes the jet kinetic energy increment Δ_j in the absence of cooling.

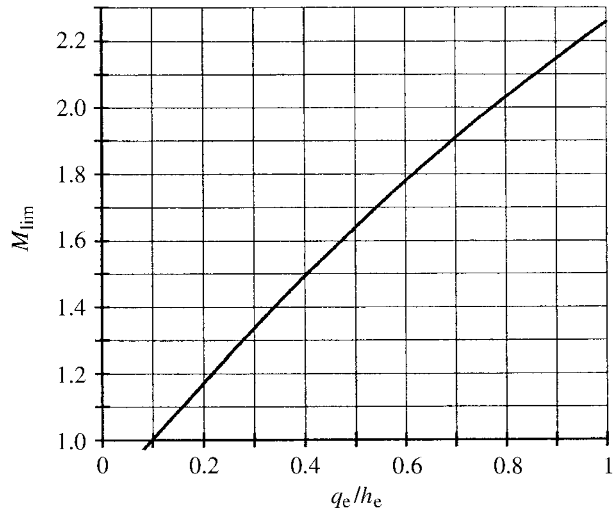

Figure 5. Variation of minimum incident Mach number $M_{\rm lim}$ of an evaporation shock as a function of the ratio of the loss of total heat through the shock $q_{\rm e}$ to the enthalpy $h_{\rm e}$ of the incident stream.

Table 1. *The least incident normal Mach number $M_{\rm lim}$ to the (first) shock that provides a given evaporative heat loss q_1*

q_1/h_∞	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$M_{\rm lim}$	1.0034	1.1713	1.3363	1.4930	1.6400	1.7780	1.9079	2.0307	2.1472	2.2583

value of $M_{\rm ns}$ for which $\Delta_{\rm j}$ is maximum in the uncooled condition (in this instance given by $q_0 = 0$). However, it is not to be inferred that this is necessarily how $M_{\rm ns}$ would be chosen in practice.

(*b*) *The choice of strength of the evaporative shock*

If the first compression is the (single) evaporative shock—and this appears to remain the most effective position for large as well as just for small values of $q_{\rm e}$—there still remains the problem of how best to select the strength of that shock. This is determined by its incident normal Mach number, $M_{\rm ne}$. In the analysis of the previous section, where $q_{\rm e}$ is regarded as infinitesimal, it was assumed that $M_{\rm ne}$ is the same as all the other shock strengths ($M_{\rm ns}$). That appears reasonable in the context, as with $q_{\rm e}/h_\infty$ small there is by definition little to distinguish the evaporative shock from any other. However, where $q_{\rm e}/h_\infty$ is not necessarily small, the matter of best choice is not obvious.

As was remarked earlier, there is a minimum evaporative shock strength associated with the existence condition that the temperature through the shock does not decrease. This limit value $M_{\rm lim}$ depends on $q_{\rm e}/h_{\rm e}$ or, in application to the first shock in a sequence and subject to our simplifying assumptions, on the value of q_1/h_∞. It is shown in figure 5 and table 1.

The significance of choosing $M_{\rm ne}$ as small as possible, and therefore equal to $M_{\rm lim}$, is that it can be shown that this minimizes the choking Mach number $M_{\rm min}$ (provided

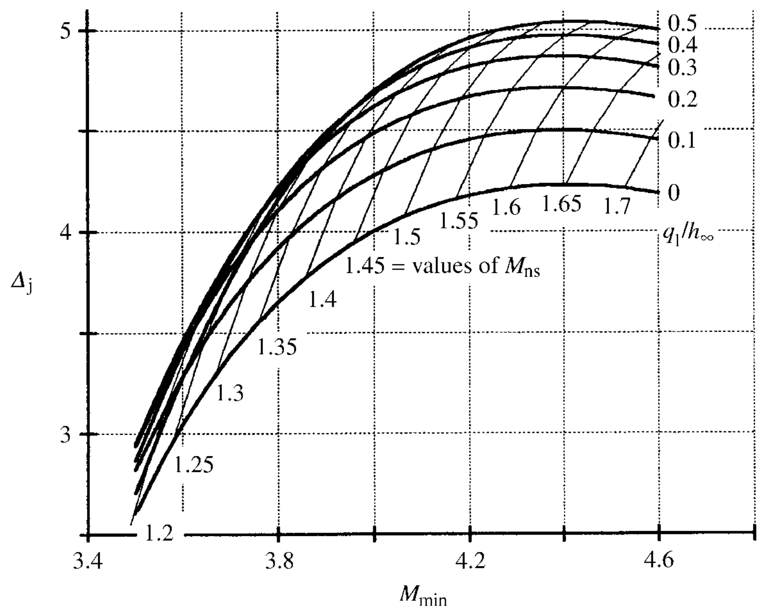

Figure 6. Effect of heat loss q_1 through an initial evaporation shock with $M_{n1} = M_{lim}$ on the kinetic energy increment Δ_j and the minimum flight Mach number M_{min}, as determined by the incident Mach number M_{ns} of the remaining three compression shocks (with $h_{max}/h_\infty = 10$).

n, M_{ns} and h_{max}/h_∞ are fixed). The effect on Δ_j and M_{min} of such a choice if the evaporative shock is the first in sequence is illustrated in figure 6. Compared with figure 1, the more limited effect of heat loss is at once apparent, but it is also clear that the relation for zero heat loss ($q_0 = 0$ and $q_1 = 0$) is different in the two cases. This seeming inconsistency is because M_{lim} is taken as unity for $q_1/h_\infty < 0.1$, so that if $q_1 = 0$, the first 'shock' degenerates to a Mach wave and the compression is simply a three-shock process, instead of the four-shock process assumed in figure 1.

Another basis for the choice of M_{ne} is that its value should maximize Δ_j if all the other parameters are held constant. We can call this value M_{opt} and its variation with q_1/h_∞ and the incident Mach number M_{ns} of the following three shocks of a four-shock system is shown in figure 7. It will be seen that its value is replaced by the least value M_{lim} if either q_e/h_∞ or M_{ns} is large. The graph of Δ_j versus M_{min} is not in this context very informative (figure 8) and we supplement it with figures 9 and 10 that show Δ_j and M_{min} plotted separately versus M_{ns}. It will be apparent that this choice of M_{ne} does not allow operation at flight Mach numbers below 4. This applies even in the limit of $q_1 \to 0$, since we see from figure 7 that although M_{ns} might be reduced—which would ordinarily allow operation at lower M_∞—the first shock in the sequence (i.e. the evaporative shock) still accords in the limit with a large M_{ne} and so a large compression. It is only to be expected that this also will serve to increase Δ_j (as is the intention) because the highest values of Δ_j are obtained with the larger compressions that result from higher M_{ns}.

A rather 'natural' choice is to make M_{ne} the same as all the other M_{ns} as we assumed in § 3 *a*. More precisely, this now means taking $M_{ne} = \max(M_{ns}, M_{lim})$, after imposing the existence condition for an evaporative shock. As is shown in figure 11, M_{ne} is replaced by the limit for low M_{ns} or high q_1. The Δ_j versus M_{min}

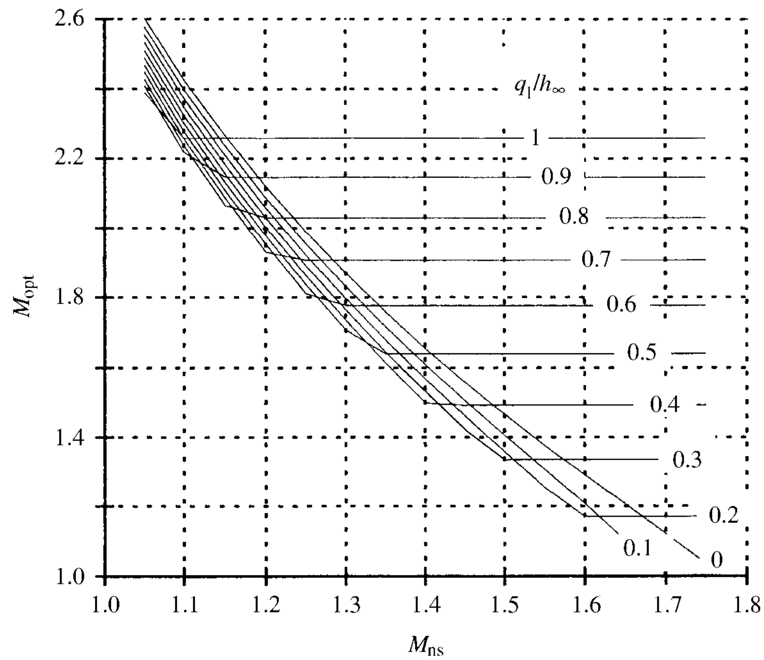

Figure 7. The 'optimum' incident Mach number M_{opt} of an evaporation shock that maximizes the kinetic energy increment Δ_{j}, in terms of the loss of total heat through the shock q_1 and the incident normal Mach number M_{ns} of the three following shocks of the four-shock system ($h_{\max}/h_\infty = 10$).

diagram (figure 12) shows that this recovers the low-Mach-number performance of the choice $M_{\text{ne}} = M_{\text{lim}}$ (figure 6), because the least $M_{\min}$ is achieved for low M_{ns}. One would therefore expect that by taking $M_{\text{n1}} = \min(M_{\text{ns}}, M_{\text{opt}})$ for the incident Mach number of the initial evaporative shock, as shown in figure 13, it should be possible to combine this desirable low-Mach-number performance with the higher values of Δ_{j} at higher M_{ns} of the optimal choice. Figure 14 shows that this is indeed so, and figures 15 and 16 provide the same detail as figures 9 and 10. Another way of exhibiting these relations is to plot the jet velocity ratio versus M_{ns} as in figures 17 and 18. This ratio bears a more direct relationship to the thrust of the scramjet. However, it varies with flight Mach number, and so lacks the generality derived from the use of the kinetic energy increment Δ_{j}, which is independent of M_∞.

(*c*) *Coolant and fuel mass flow*

Assuming this last-mentioned choice of M_{n1}, figure 19 shows the gain in total heat ΔH that has to be supplied by the combustion process. It will be seen that this becomes independent of q_1, and varies only with the enthalpy change produced by the rest of the compression shock system, if $q_1/h_\infty > 0.35$. This is because for these higher values of q_1, as figure 13 shows, the enthalpy loss across the evaporation shock becomes limited (to zero). On the one hand, the enthalpy downstream of this shock then equals that in the intake; while, on the other hand, the enthalpy is fixed (and equal to $h_{\max}$) after combustion is complete.

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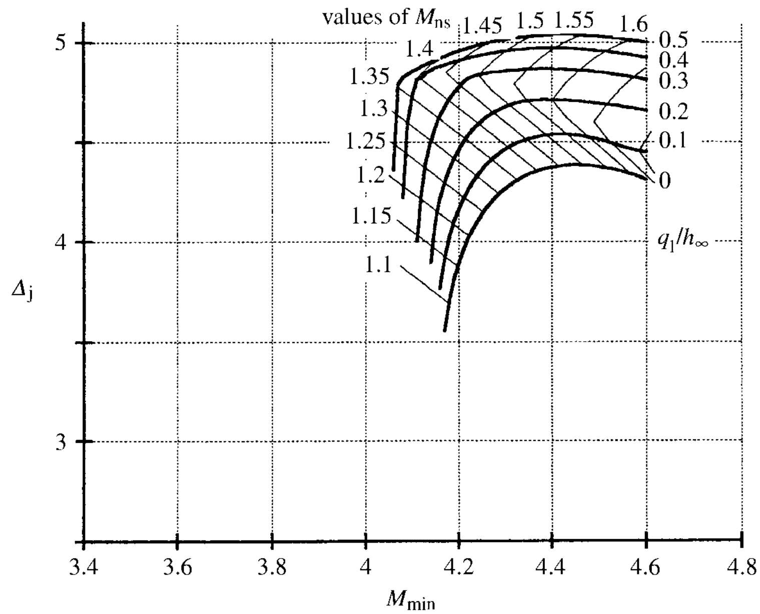

Figure 8. Effect of heat loss q_1 through an initial evaporation shock with $M_{n1} = M_{opt}$ on the kinetic energy increment Δ_j and the minimum flight Mach number M_{min}, as determined by the incident Mach number M_{ns} of the remaining three compression shocks (with $h_{max}/h_\infty = 10$).

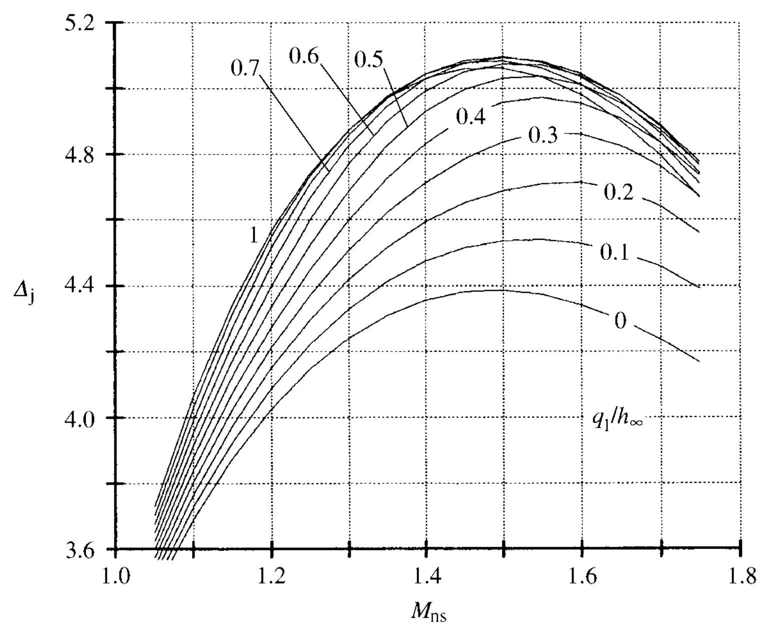

Figure 9. Maximum kinetic energy increment Δ_j for optimum $M_{ne} = M_{opt}$ as a function of M_{ns} and for various q_1/h_∞ (with $n = 4$, $e = 1$ and $h_{max}/h_\infty = 10$).

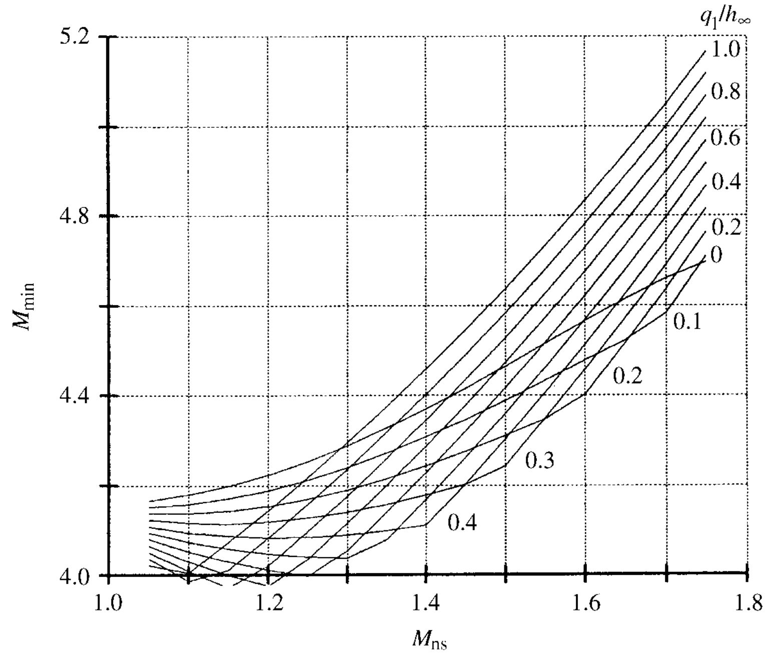

Figure 10. Minimum Mach number for choked flow with optimum $M_{\rm ne} = M_{\rm opt}$ as a function of $M_{\rm ns}$ and for various q_1/h_∞ (with $n = 4$, $e = 1$ and $h_{\max}/h_\infty = 10$).

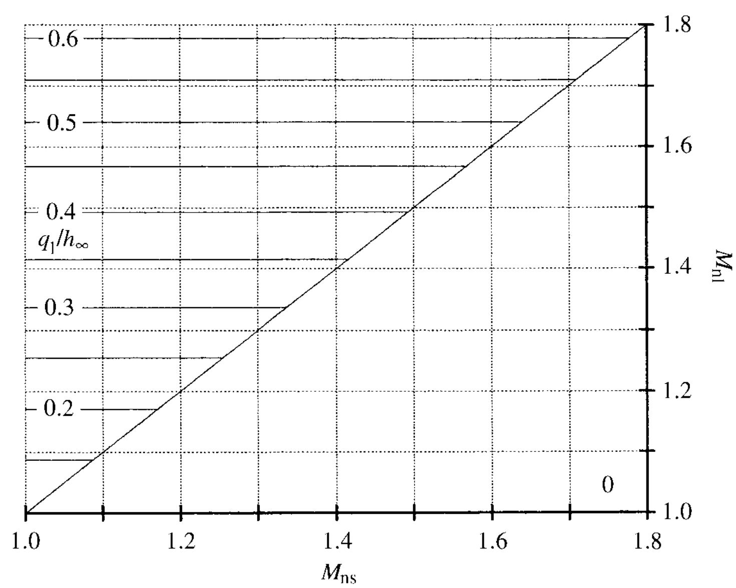

Figure 11. Variation of $M_{\rm n1} = \max(M_{\rm ns}, M_{\rm lim})$ in terms of the loss of total heat through the shock q_1 and the incident Mach number $M_{\rm ns}$ of the three following shocks of the four-shock system ($h_{\max}/h_\infty = 10$).

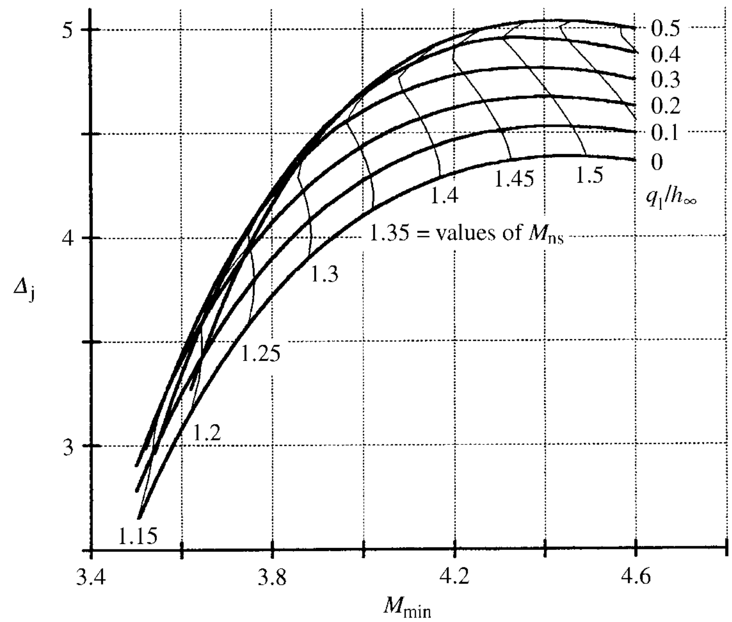

Figure 12. Effect of heat loss q_1 through an initial evaporation shock with $M_{n1} = M_{ns}$ on the kinetic energy increment Δ_j and the minimum flight Mach number M_{min}, as determined by the incident Mach number M_{ns} of the four-shock system ($h_{max}/h_\infty = 10$).

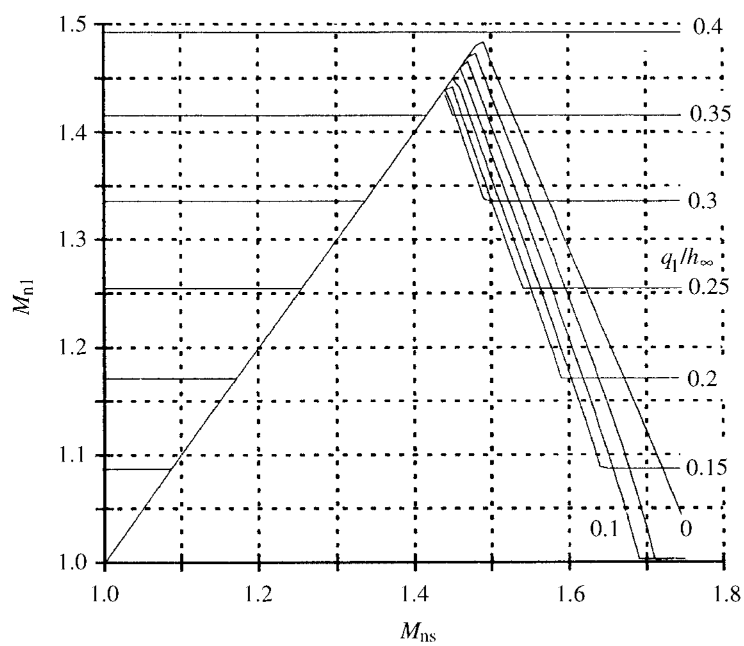

Figure 13. Variation of $M_{n1} = \min(M_{ns}, M_{opt})$ in terms of the loss of total heat through the shock q_1 and the incident Mach number M_{ns} of the three following shocks of the four-shock system ($h_{max}/h_\infty = 10$).

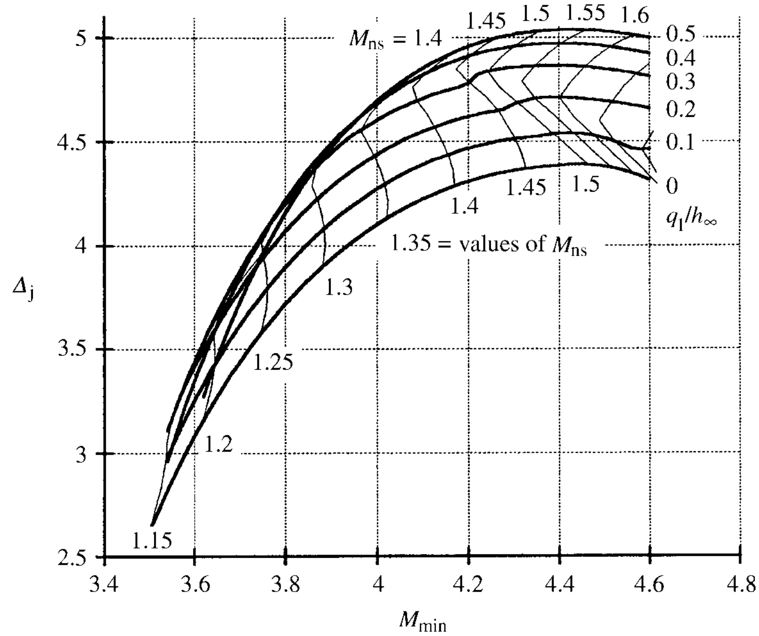

Figure 14. Effect of heat loss q_1 through an initial evaporation shock with $M_{n1} = \min(M_{ns}, M_{opt})$ on the kinetic energy increment Δ_j and the minimum flight Mach number M_{min}, as determined by the incident Mach number M_{ns} of the four-shock system ($h_{max}/h_\infty = 10$).

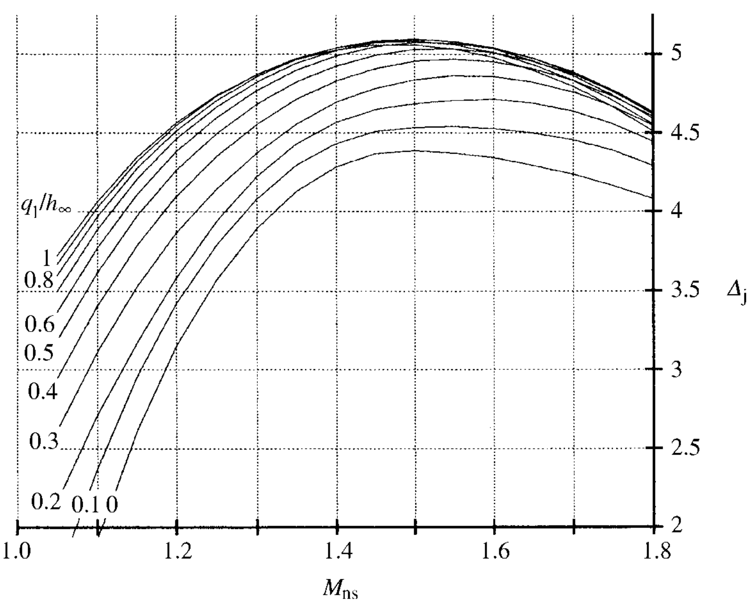

Figure 15. Maximum kinetic energy increment Δ_j for $M_{ne} = \min(M_{ns}, M_{opt})$ as a function of M_{ns} and for various q_1/h_∞ (with $n = 4$, $e = 1$ and $h_{max}/h_\infty = 10$).

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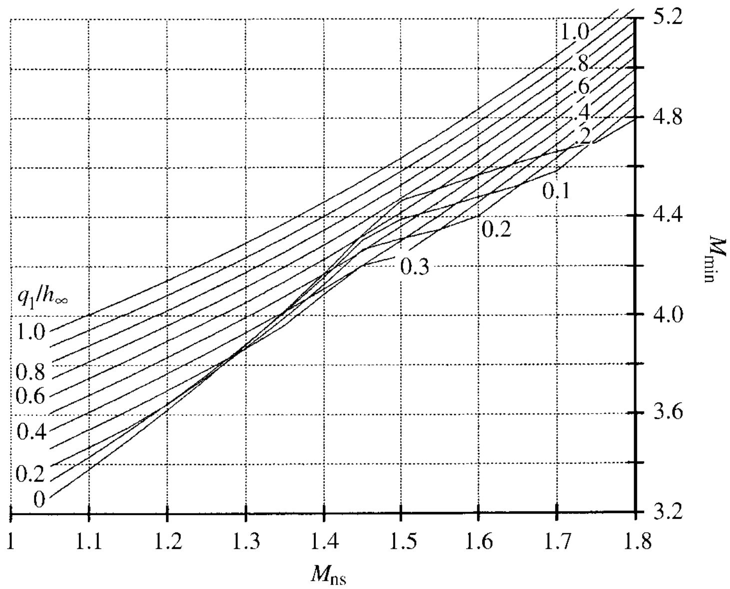

Figure 16. Minimum Mach number for choked flow with $M_{ne} = \min(M_{ns}, M_{opt})$ as a function of M_{ns} and for various q_1/h_∞ (with $n = 4$, $e = 1$ and $h_{max}/h_\infty = 10$).

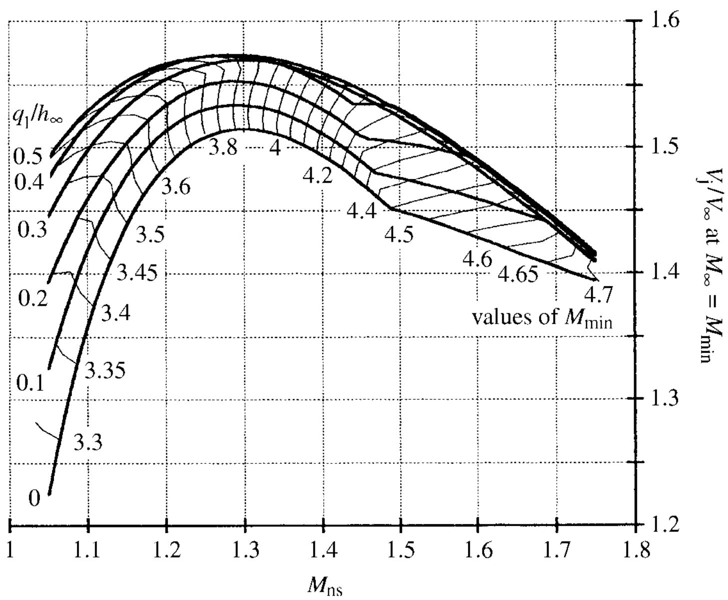

Figure 17. Values of M_{min} and the jet velocity ratio V_j/V_∞ at $M_\infty = M_{min}$ as affected by the heat loss q_1 through an initial evaporative shock with $M_{n1} = \min(M_{ns}, M_{opt})$ and by the incident Mach number M_{ns} of the remaining three shocks ($h_{max}/h_\infty = 10$).

If we ignore any thermochemical reaction of the coolant in the combustion process, the fuel mass flow is simply $w_a \Delta H/c$, where w_a is the air mass flow and c is the calorific value of the fuel. Hence if we express the coolant mass flow as μw_a, the

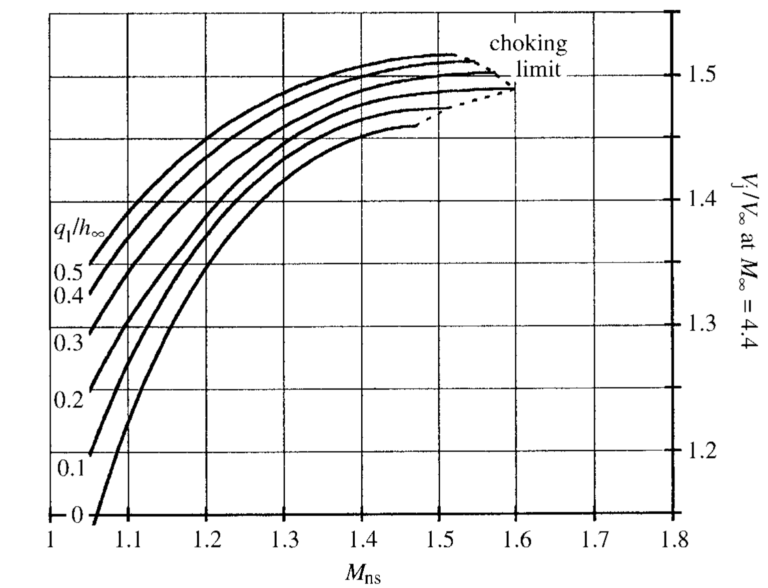

Figure 18. Values of jet velocity ratio V_j/V_∞ at $M_\infty = 4.4$ as affected by the heat loss q_1 through an initial evaporative shock with $M_{n1} = \min(M_{ns}, M_{opt})$ and by the incident Mach number M_{ns} of the remaining three shocks ($h_{max}/h_\infty = 10$).

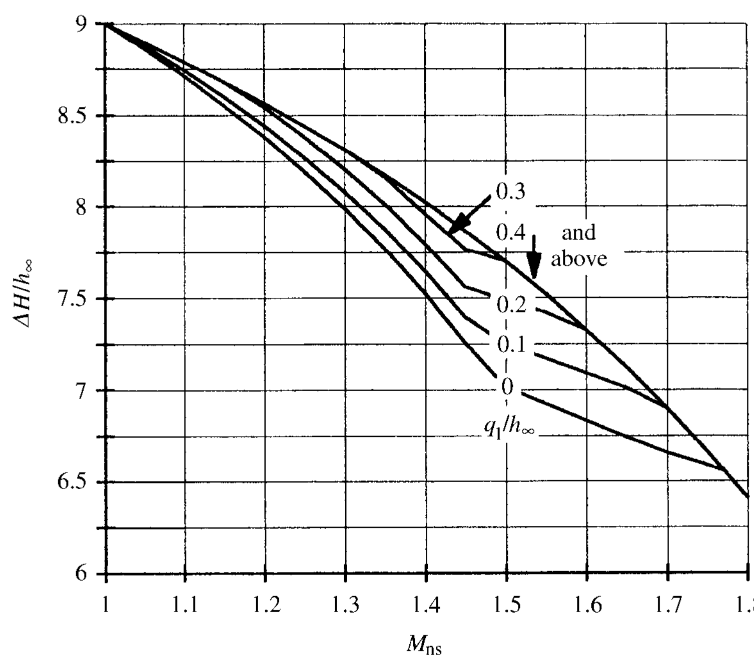

Figure 19. Heat added during combustion ΔH as a multiple of h_∞ in terms of M_{ns} for various q_1/h_∞, assuming $M_{ne} = \min(M_{ns}, M_{opt})$, $n = 4$, $e = 1$ and $h_{max}/h_\infty = 10$.

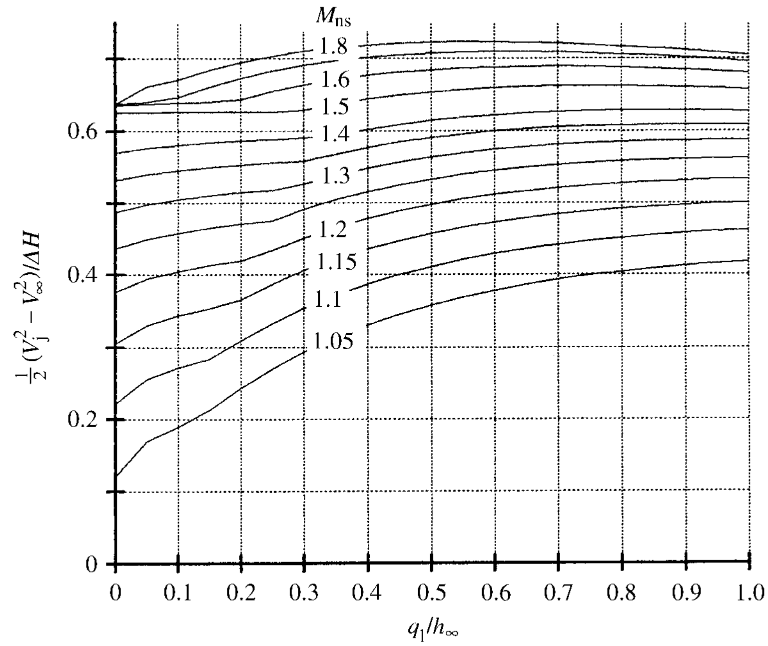

Figure 20. Energy ratio $\frac{1}{2}(V_j^2 - V_\infty^2)/\Delta H$ in terms of q_1/h_∞ for various M_{ns}, assuming $M_{ne} = \min(M_{ns}, M_{opt})$, $n = 4$, $e = 1$ and $h_{max}/h_\infty = 10$.

ratio of coolant to fuel is $\Gamma = \mu c/\Delta H$. For a typical hydrocarbon fuel, c would be *ca.* 5×10^8 ft^2 s^{-2} (which is *ca.* 20 000 Btu lb^{-1} of fuel, or 1.5×10^7 ft lbf lb^{-1}). Thus for flight in the stratosphere, $\Gamma \approx 200\mu(h_\infty/\Delta H)$. The heat loss q_1 due to the coolant flow is also proportional to μ and placing $q_1/h_\infty = b\mu$ say, the ratio of coolant to fuel is $\Gamma \approx (200/b)(h_\infty/\Delta H)(q_1/h_\infty)$. If the coolant is ammonia, the value of b can be taken to be about 7, and so from figure 19 we deduce that the coolant mass flow is between about three and four times that of the fuel (depending mainly on M_{ns}) for unit values of q_1/h_∞.

This makes it clear that for precooling to have a significant effect, the coolant mass flow is likely to be appreciable compared with the fuel consumption. This would be even more marked if the fuel were hydrogen, since its calorific value is about 2.5 times that assumed for the hydrocarbon, and the fuel mass flow would be proportionately reduced. Precooling therefore seems only likely to be considered for transient, rather than continuous, operation unless of course it provides a commensurate decrease in specific fuel consumption (β, say). That does not, however, seem to be the case generally.

Figure 20 shows values of $\frac{1}{2}(V_j^2 - V_\infty^2)/\Delta H$, which are related directly to the specific (fuel) impulse, $I = 1/\beta$. This is because $(V_j - V_\infty)$ is small compared with V_∞ at high Mach numbers, and so the thrust T is $w_a(V_j - V_\infty) \approx \frac{1}{2}(V_j^2 - V_\infty^2)w_a/V_\infty$. As we have seen, the fuel mass flow is $w_f = w_a\Delta H/c$ and since I is equal to T/w_f, the specific impulse is therefore approximately (c/gV_∞) times the non-dimensional value of $\frac{1}{2}(V_j^2 - V_\infty^2)/\Delta H$. It will be clear from the figure that an increase of q_1/h_∞ in general increases the value of $\frac{1}{2}(V_j^2 - V_\infty^2)/\Delta H$, and so increases I. Even so, it is only at the lowest values of M_{ns} (and so the lowest M_∞) that the consequent reduction in fuel consumption is likely to offset the additional increase of coolant

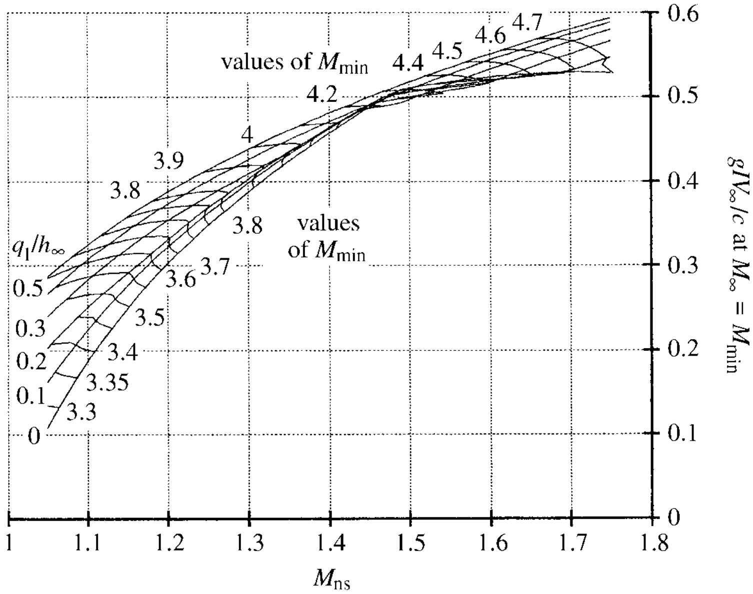

Figure 21. Values of $M_{\min}$ and gIV_∞/c at $M_\infty = M_{\min}$, where I is the specific (fuel) impulse, as affected by the heat loss q_1 through an initial evaporative shock with $M_{n1} = \min(M_{ns}, M_{opt})$ and by the incident Mach number M_{ns} of the remaining three shocks ($h_{\max}/h_\infty = 10$).

mass flow. In this connection, we note that what can be termed the specific *propellant* consumption would equal $(1+\Gamma)\beta$, so as to include both the mass flow of the coolant and that of the fuel. Correspondingly, the specific propellant impulse would be simply the inverse and so equal to $I/(1+\Gamma)$.

To convert the values shown in figure 20 to a measure of specific impulse, we note that for a typical hydrocarbon fuel, c/gV_∞ would be *ca.* 1500 s at a flight Mach number of 10. This therefore is the specific fuel impulse corresponding to a unit value of $\frac{1}{2}(V_j^2 - V_\infty^2)/\Delta H$ and it is equivalent to a specific fuel consumption β of *ca.* 2.5 lb h^{-1} of fuel per lbf thrust. Our simplified algebra is less suitable as a means of estimating I at much lower Mach numbers than this, but figures 21 and 22 show values of (gIV_∞/c) for M_∞ equal to $M_{\min}$ and 4.4, to match figures 17 and 18.

(*d*) *The choice of coolant*

Without regard to the physical properties of any particular coolant, our analysis has shown that it is advantageous to introduce it as early as possible in the flow process. Further, if any substantial loss of total heat is achieved through an initial evaporation shock, then the temperature rise through the shock is best taken as limited (i.e. almost zero). It follows that ideally the coolant should start to evaporate in the intake flow, so that it meets and leaves the first shock at a temperature equal to its boiling point.

In our simplified model, the intake flow is supposed to have the same conditions as the free stream. Assuming flight in the stratosphere, this would suggest that liquid ammonia would be well suited for use. The boiling point (T_b) of a 5% mixture by mass of NH_3 in air corresponding to a range of flight conditions is tabulated above

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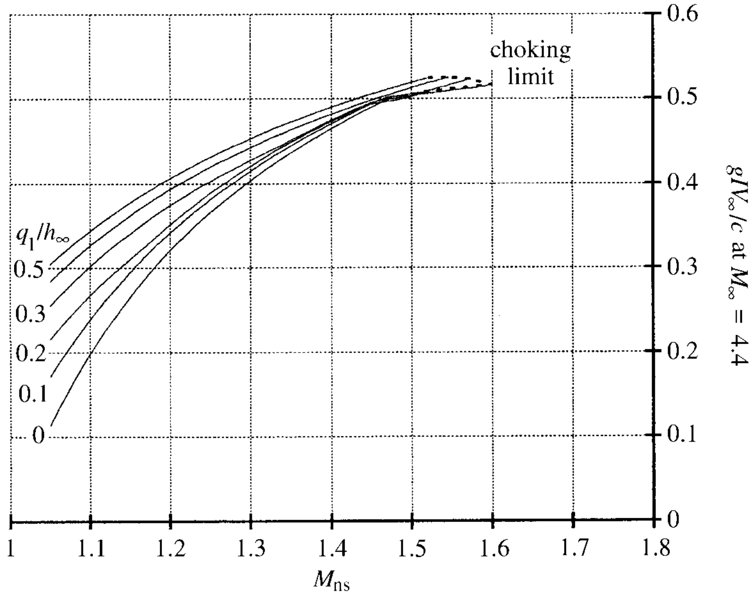


Figure 22. Values of gIV_∞/c at $M_\infty = 4.4$, where I is the specific (fuel) impulse, as affected by the heat loss q_1 through an initial evaporative shock with $M_{\rm n1} = \min(M_{\rm ns}, M_{\rm opt})$ and by the incident Mach number $M_{\rm ns}$ of the remaining three shocks ($h_{\max}/h_\infty = 10$).

Table 2. *The boiling point of ammonia in a range of free-stream conditions*

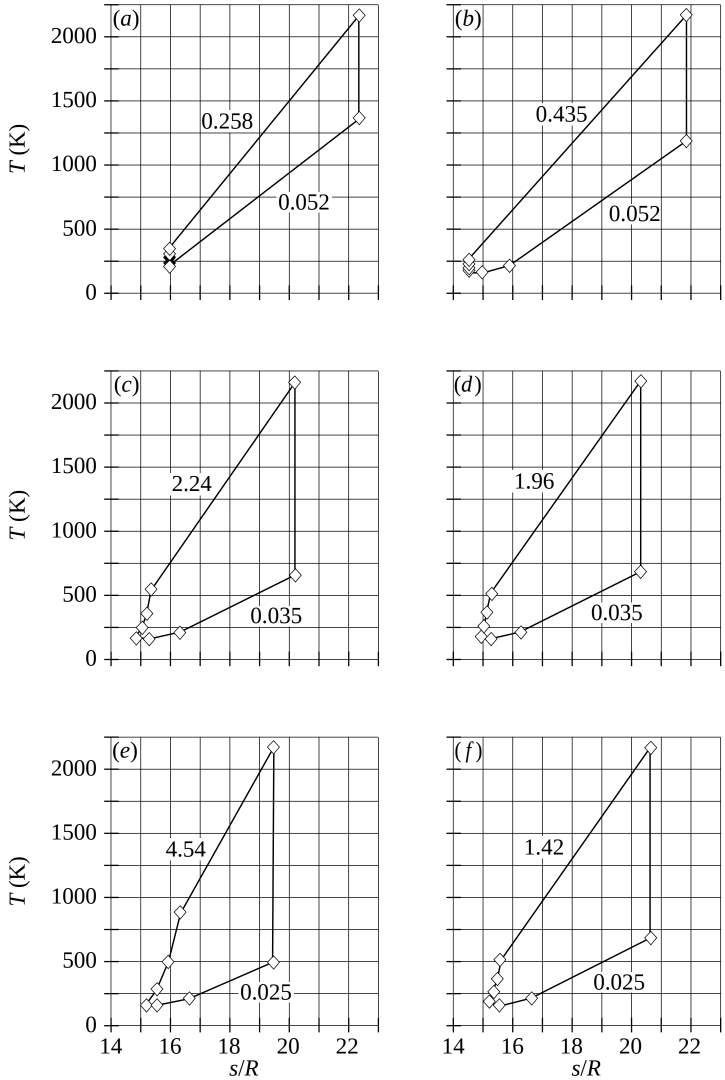
	$\frac{1}{2}\rho_\infty V_\infty^2 = 1000$ lbf ft^{-2}			$\frac{1}{2}\rho_\infty V_\infty^2 = 2000$ lbf ft^{-2}		
	$M_\infty = 3.6$	$M_\infty = 4.4$	$M_\infty = 5.2$	$M_\infty = 3.6$	$M_\infty = 4.4$	$M_\infty = 5.2$
p_∞ (atm)	0.0521	0.0349	0.025	0.1042	0.0697	0.050
$T_{\rm b}$ (K)	166.4	162.8	159.9	173.1	169.2	166.1
μ_0	0.0331	0.0355	0.0374	0.0287	0.0313	0.0334

(see table 2). In each instance, the boiling point is substantially below the likely minimum atmospheric temperature in the stratosphere.

To keep within the framework of our calculations, it would therefore be necessary to consider that a proportion of the ammonia, equal to a fraction μ_0 of the air mass flow (as given in table 2), is evaporated in the intake at constant pressure. This would be sufficient to bring the temperature of the air–vapour mixture down to the boiling point of the liquid NH_3. The remaining mass fraction of the liquid (μ_1, say) converts into a heat loss $q_1 \approx 7\mu_1 h_\infty$ across the initial evaporation shock.

Figures 23 and 24 show a couple of entropy–temperature diagrams of the assumed (simplified) scramjet cycle for each of the six intake conditions shown in table 2. These diagrams include the precooling before the initial evaporative shock, and assume that $\mu_0 + \mu_1 = 0.05$ (except for the two examples with $M_\infty = 3.6$ and $\mu_0 = \mu_1 = 0$). The incident normal Mach number of the evaporative shock is chosen as in figure 13.

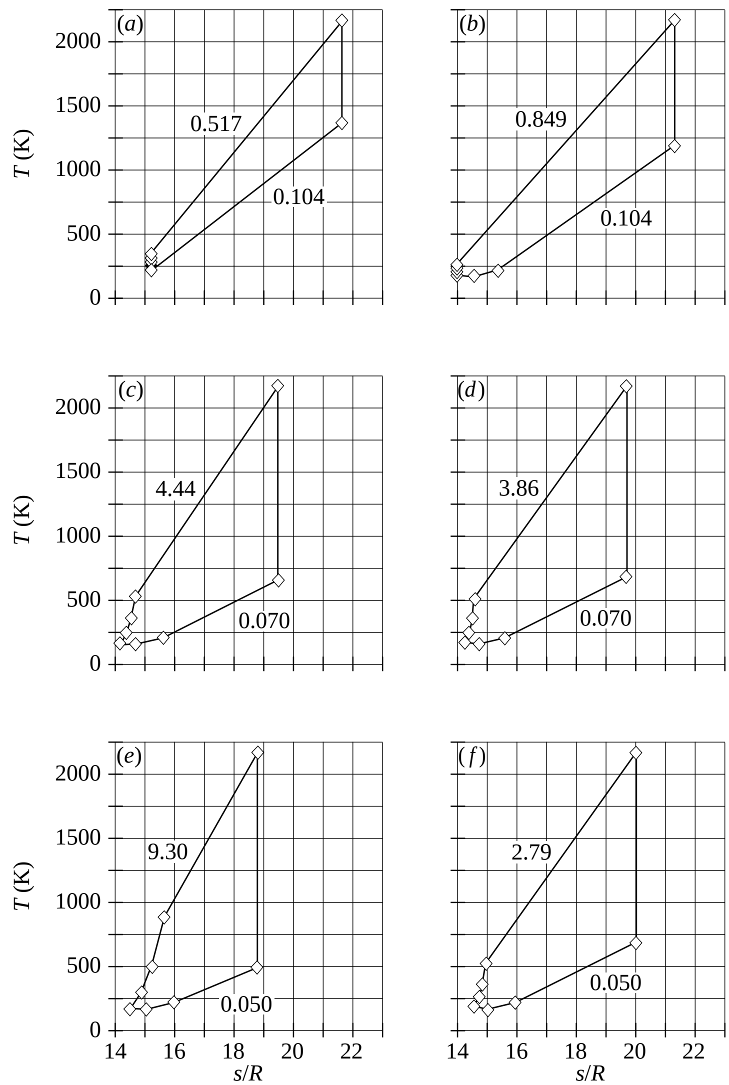
In reality, the intake would be likely to be within the airflow that is compressed and heated by the shock waves generated by the aircraft. The amount of evaporation



	M	$M_{\min}$	M_{n}	q_1/h_∞	V_{j}/V_∞		M	$M_{\min}$	M_{n}	q_1/h_∞	V_{j}/V_∞
(*a*)	3.6	3.6	1.1925	0	1.4790	(*b*)	3.6	3.6	1.2344	0.1181	1.5891
(*c*)	4.4	4.4	1.7286	0.1014	1.5210	(*d*)	4.4	—	1.6221	0.1014	1.5235
(*e*)	5.2	5.2	2.0935	0.0881	1.3419	(*f*)	5.2	—	1.6181	0.0881	1.3954

Figure 23. Some entropy (s) versus temperature (T) charts of the scramjet cycle for choking ($M = M_{\min}$) and maximum jet velocity conditions ($\frac{1}{2}\rho_\infty V_\infty^2 = 1000$ lbf ft^{-2}). (R is the gas constant of air and the numbers against the isobars represent pressure in atmospheres.)

that would take place before the initial shock of the ramjet compression would then be higher. There might be other coolants that then come into contention. Indeed, there could be contexts in which the temperature becomes high enough to boil water, which yields a proportionately larger heat loss ($q_{\mathrm{e}}/h_\infty \approx 10\mu$).



	M	$M_{\min}$	$M_{\rm n}$	q_1/h_∞	$V_{\rm j}/V_\infty$		M	$M_{\min}$	$M_{\rm n}$	q_1/h_∞	$V_{\rm j}/V_\infty$
(*a*)	3.6	3.6	1.1925	0	1.4790	(*b*)	3.6	3.6	1.2267	0.1490	1.5847
(*c*)	4.4	4.4	1.7274	0.1310	1.5201	(*d*)	4.4	—	1.6312	0.1310	1.5221
(*e*)	5.2	5.2	2.0671	0.1167	1.3430	(*f*)	5.2	—	1.6268	0.1167	1.3944

Figure 24. Some entropy (s) versus temperature (T) charts of the scramjet cycle for choking ($M = M_{\min}$) and maximum jet velocity conditions ($\frac{1}{2}\rho_\infty V_\infty^2 = 2000$ lbf ft^{-2}). (R is the gas constant of air and the numbers against the isobars represent pressure in atmospheres.)

4. Conclusions

Our study suggests a limited although significant increase in thrust resulting from the use of a precoolant. With appropriate design, this increase in thrust can also be expected at flight Mach numbers down to 3.5. Therefore, precooling may extend the use of a scramjet to lower flight Mach numbers. There is also a resulting increase in specific fuel impulse associated with the precooling, especially at the lowest flight Mach numbers. However, the specific propellant impulse (including the coolant flow) is generally decreased.

To achieve the greatest effect, the coolant should be allowed to evaporate in the intake and/or across the first shock of the compression. For flight in the stratosphere at Mach numbers around 4 or 5, liquid ammonia would seem to be particularly suited to such a role (and it has other qualities to recommend it).

It is felt that the problem deserves a more realistic approach, targeted to a particular coolant, than our simplified, but quite general, analysis could be expected to provide (and indeed more refined calculation has confirmed that view, as shown in figure 3, p. 2323).

Appendix A. Analysis of the 'basic' engine cycle

The jet velocity is given by

$$V_{\mathrm{j}}^2 = 2(H_{\mathrm{j}} - h_{\mathrm{j}}), \tag{A 1}$$

where H is the total enthalpy and h is the local specific enthalpy. If the jet is assumed to be expanded isentropically to atmospheric pressure, then since H_{j} is equal to H_{pc}, the post-combustion total enthalpy

$$V_{\mathrm{j}}^2 = 2[H_{\mathrm{pc}} - (p_\infty/p_{\mathrm{pc}})^{(\gamma-1)/\gamma} h_{\mathrm{pc}}]. \tag{A 2}$$

We assume further that combustion is a constant-pressure (and so constant-velocity) process and that h_{pc} is specified as h_{max}. Then if subscript 'comp' denotes conditions after the end of compression and before combustion,

$$H_{\mathrm{pc}} - H_{\mathrm{comp}} = h_{\mathrm{max}} - h_{\mathrm{comp}}, \tag{A 3}$$

$$p_{\mathrm{pc}} = p_{\mathrm{comp}}, \tag{A 4}$$

and so

$$V_{\mathrm{j}}^2 = 2\{H_{\mathrm{comp}} - h_{\mathrm{comp}} + [1 - (p_\infty/p_{\mathrm{comp}})^{(\gamma-1)/\gamma}] h_{\mathrm{max}}\}. \tag{A 5}$$

We assume next that the compression is an n-shock process, it being allowed that there is a loss of total heat q_k across the kth shock, so that $H_{k+1} = H_k - q_k$. We suppose that each shock supports a pressure ratio $\varpi_k = p_{k+1}/p_k = \varpi(m_k, \delta_k)$, and an enthalpy ratio $\theta_k = h_{k+1}/h_k = \theta(m_k, \delta_k)$ for $k = 1, 2, \ldots n$, where we have placed $\delta_k = q_k/h_k$ and $m_k = M_{\mathrm{n}k}^2 - 1$, $M_{\mathrm{n}k}$ being the incident normal Mach number to the kth shock. Then,

$$h_{\mathrm{comp}} = h_1 \theta_1 \theta_2 \cdots \theta_n = h_1 \prod_{k=1}^{n} \theta_k \tag{A 6}$$

and similarly

$$p_{\mathrm{comp}} = p_1 \varpi_1 \varpi_2 \cdots \varpi_n = p_1 \prod_{k=1}^{n} \varpi_k. \tag{A 7}$$

Hence in (A 5)

$$V_{\mathrm{j}}^2 = 2\left\{H_1 - \sum_{k=1}^{n} q_k - h_1 \prod_{k=1}^{n} \theta_k + \left[1 - \left(\frac{p_\infty}{p_1}\right)^{(\gamma-1)/\gamma} \prod_{k=1}^{n} \varpi_k^{(1-\gamma)/\gamma}\right] h_{\max}\right\}. \quad \text{(A 8)}$$

To complete the cycle, we must consider the injection of the coolant into the intake, where we assume that the pressure is again equal to p_∞. We ignore any change in mass or total momentum flux due to the coolant addition. Only the effect of a (possible) change in enthalpy is considered. We therefore ignore both the momentum exchange and the mixing losses. Assuming further that cooling at constant pressure takes place ahead of the first shock, it follows that $p_1 = p_\infty$ and $H_1 - H_\infty = h_1 - h_\infty = -q_0$. Then placing

$$\varDelta_{\mathrm{j}} = \tfrac{1}{2}(V_{\mathrm{j}}^2 - V_\infty^2)/h_\infty, \quad \text{(A 9)}$$

so that

$$\frac{V_{\mathrm{j}}}{V_\infty} = \sqrt{1 + \frac{2\varDelta_{\mathrm{j}}}{(\gamma-1)M_\infty^2}}, \quad \text{(A 10)}$$

it follows from (A 8), since $H_\infty = h_\infty + \frac{1}{2}V_\infty^2$, that

$$\varDelta_{\mathrm{j}} = 1 - \sum_{k=0}^{n} \frac{q_k}{h_\infty} - \left(1 - \frac{q_0}{h_\infty}\right) \prod_{k=1}^{n} \theta_k + \left(1 - \prod_{k=1}^{n} \varpi_k^{(1-\gamma)/\gamma}\right)\left(\frac{h_{\max}}{h_\infty}\right). \quad \text{(A 11)}$$

However, neither (A 10) nor (A 11) can be applied for such low values of M_∞ that the combustion process is choked. In (A 2) we see that for the flow to leave the combustion process at supersonic speed, H_{pc} must be at least $\frac{1}{2}(\gamma+1)h_{\mathrm{pc}} = \frac{1}{2}(\gamma+1)h_{\max}$.

Taking the shock-wave relations as those appropriate to a perfect gas,

$$\varpi(m,\delta) = 1 + \gamma\phi(m,\delta), \quad \text{(A 12)}$$

$$\frac{\theta(m,\delta)}{\varpi(m,\delta)} = 1 - \frac{\phi(m,\delta)}{m+1}, \quad \text{(A 13)}$$

where

$$\phi(m,\delta) = \frac{\sqrt{[m^2 + 2(\gamma+1)(m+1)\delta]} + m}{\gamma+1}. \quad \text{(A 14)}$$

Where there is an evaporative heat loss, it is implicit that the shock increases the temperature of the gas sufficiently to cause evaporation. It is therefore realistic to impose the condition that the enthalpy at least does not decrease across the shock (i.e. $\theta \geqslant 1$). This condition may be shown to imply that the shock must be sufficiently strong that the normal Mach number $M_{\mathrm{n}} = \sqrt{m+1}$ exceeds the limit given by

$$(\gamma-1)M_{\mathrm{n}}^2 \geqslant \sqrt{\{\delta^2 + [(\gamma-1)/\gamma]^2\}} + \delta, \quad \text{(A 15)}$$

or equivalently, that the heat loss can be no more than that implied by

$$\delta \leqslant \tfrac{1}{2}(\gamma-1)\left[M_{\mathrm{n}}^2 - \frac{1}{(\gamma M_{\mathrm{n}})^2}\right]. \quad \text{(A 16)}$$

If we regard δ as infinitesimal, then in (A 14),

$$\phi(m,\delta) \approx \frac{2m}{\gamma+1} + (1+m^{-1})\delta. \tag{A 17}$$

In particular, for a simple shock wave with no evaporative heat loss (i.e. with $\delta = 0$), we see that $\phi \equiv 2m/(\gamma+1)$. Placing $\varpi(m,0) = \varpi(m)$ and $\theta(m,0) = \theta(m)$ and assuming that all the m_k are the same and all the q_{e} are infinitesimal for $k = 1,2,\dots,n$ it can be shown after some considerable algebra that

$$\frac{1}{2}\frac{\partial V_{\mathrm{j}}^2}{\partial q_{\mathrm{e}}} = (\gamma-1)\frac{(Cm-1)\theta^{n-1}(m) + (m+1)\varpi^{n(1-\gamma)/\gamma-1}(m)(h_{\max}/h_\infty)}{m\theta^{e-1}(m)} - 1, \tag{A 18}$$

where $C = \gamma(3-\gamma)/(\gamma^2-1)$. As e only enters into the term θ^{e-1} in the denominator, we see that the thrust (if any) due to precooling *decreases* the further *downstream* the coolant is introduced. In particular, if m is chosen to maximize Δ_{j}, which can be shown to imply that

$$\theta(m)^{n-1}\varpi(m)^{n(\gamma-1)/\gamma+1}[\gamma + (m+1)^{-2}] = (\gamma+1)(h_{\max}/h_\infty), \tag{A 19}$$

then

$$\frac{1}{2}\frac{\partial V_{\mathrm{j}}^2}{\partial q_{\mathrm{e}}} = \frac{\{\gamma(3-\gamma) - (\gamma^2-1)m^{-1} + (\gamma-1)(1+m^{-1})[\gamma+(m+1)^{-2}]\}\theta^{n-e}(m)}{\gamma+1} - 1, \tag{A 20}$$

which will be seen to depend on $(n-e)$, rather than separately on n and e.

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